University of Ioannina
Department of Materials Science & Engineering
Computational Materials Science

Ηλεκτρικές και οπτικές και ιδιότητες γραφενίου

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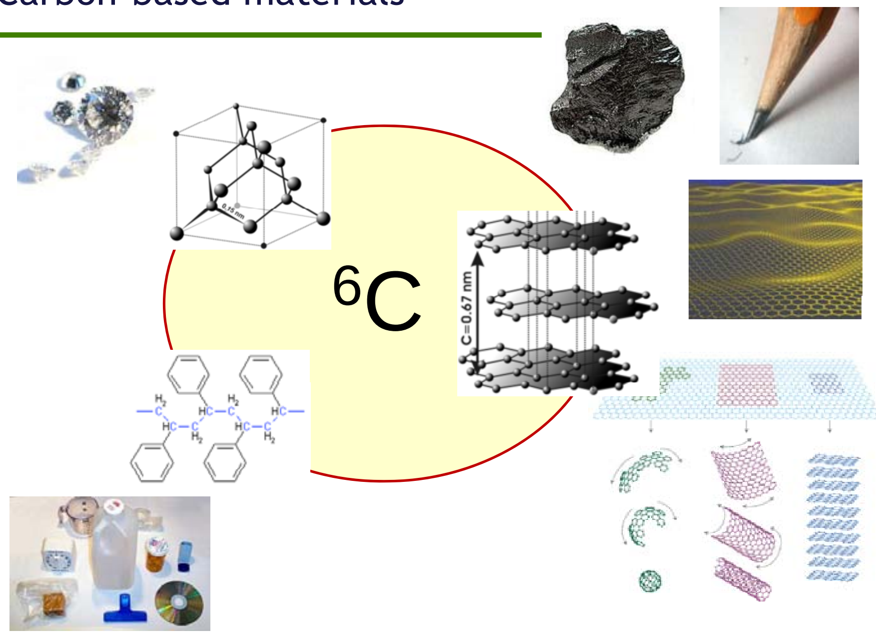
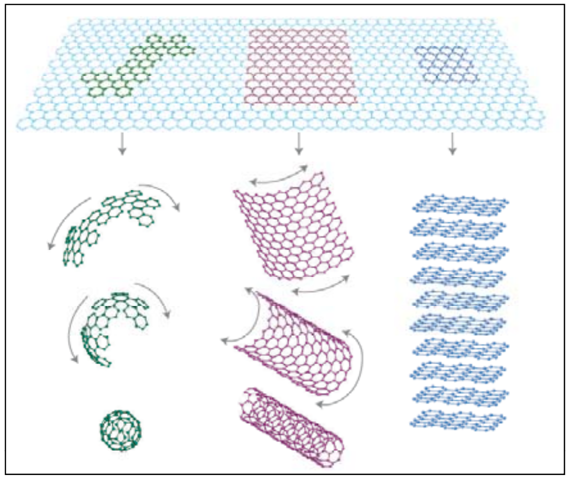
Carbon-based materials



Graphene

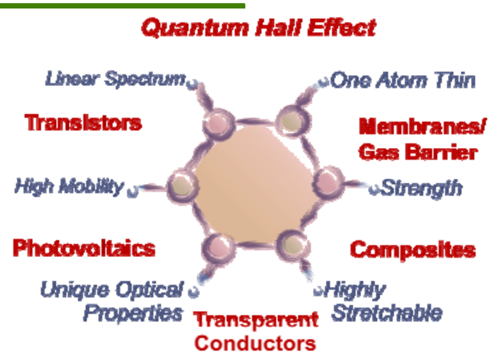
- a one-atom-thick planar sheet of sp^2-bonded carbon atoms



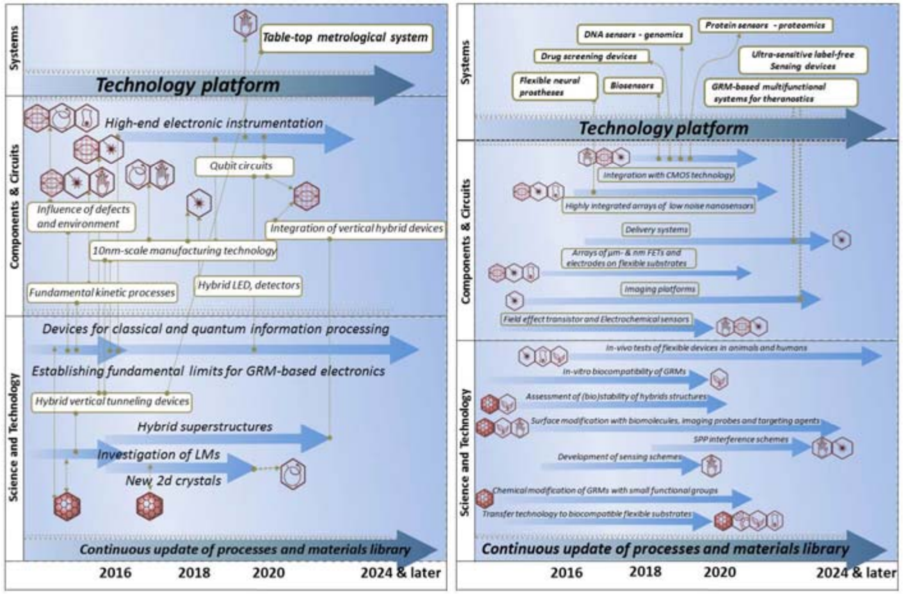
Picture from: A.K. Geim and K.S. Novoselov, "The rise of graphene", Nat. Mater. 6, 183 (2007)

Graphene applications

- Fundamental research
- Health and environment
- Production
- Electronic devices
- Spintronics
- Photonics and Optoelectronics
- Sensors
- Flexible electronics
- Energy storage and generation
- Composites
- Biomedical applications

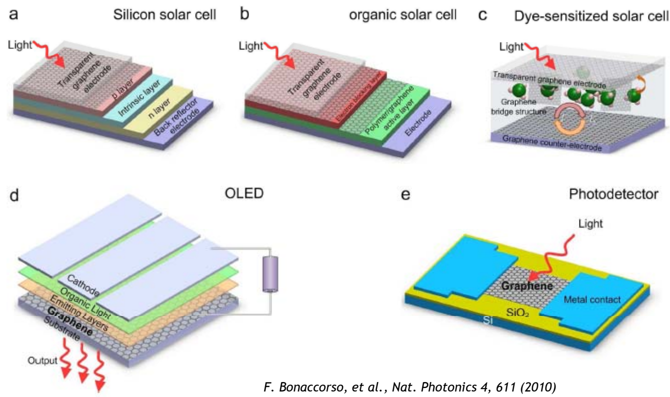


Graphene roadmap http://graphene-flagship.eu/wp-content/uploads/2013/11/simpleshow_EN_Graphene_131004-640x360.mp4



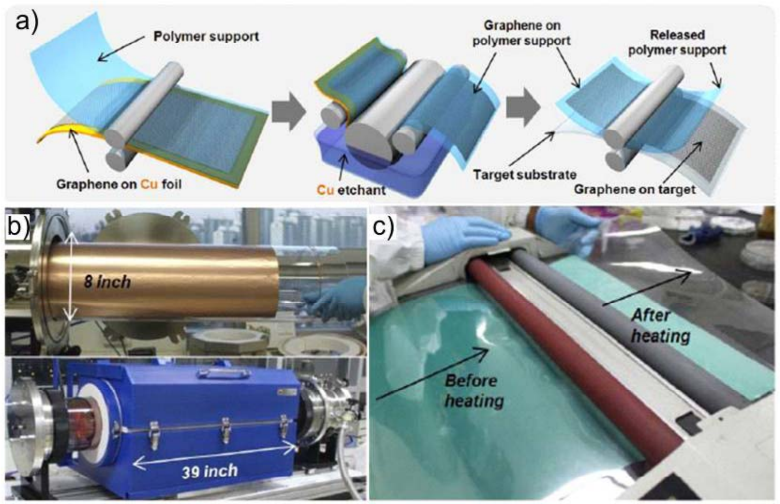
Some of graphene's optoelectronic applications

- Great properties for optoelectronics
  - High mobility - optical transparency – flexibility – environmental stability



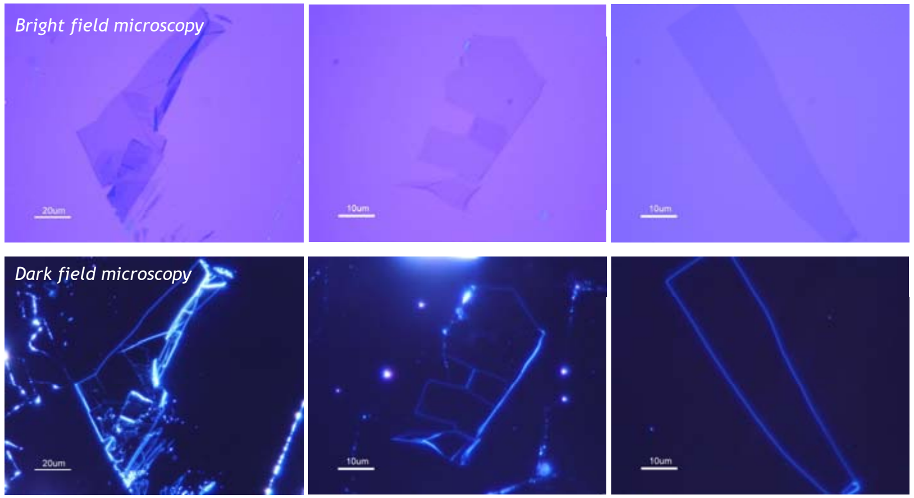
F. Bonaccorso, et al., Nat. Photonics 4, 611 (2010)

Growing graphene



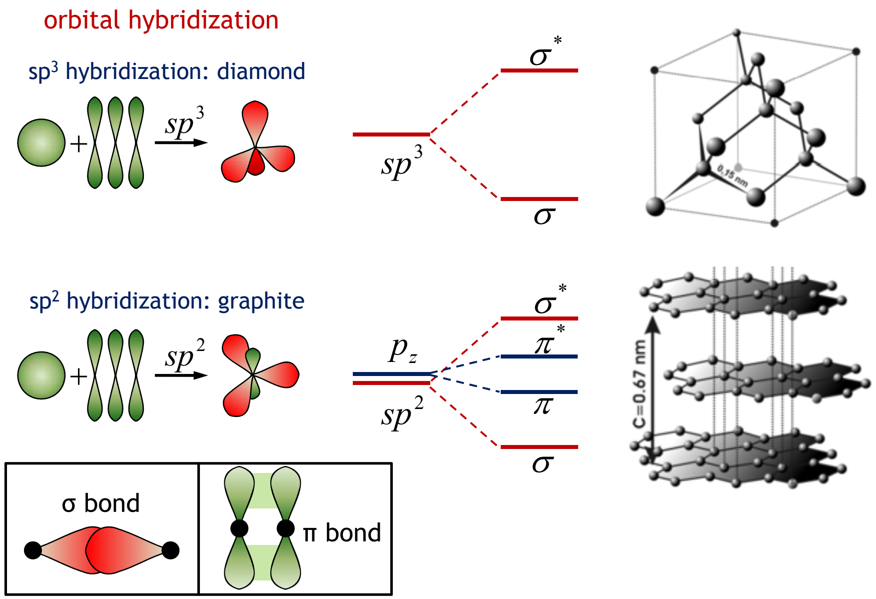
How does graphene look like?

A single graphene layer of thickness d=0.335 nm is visible to the naked eye!!

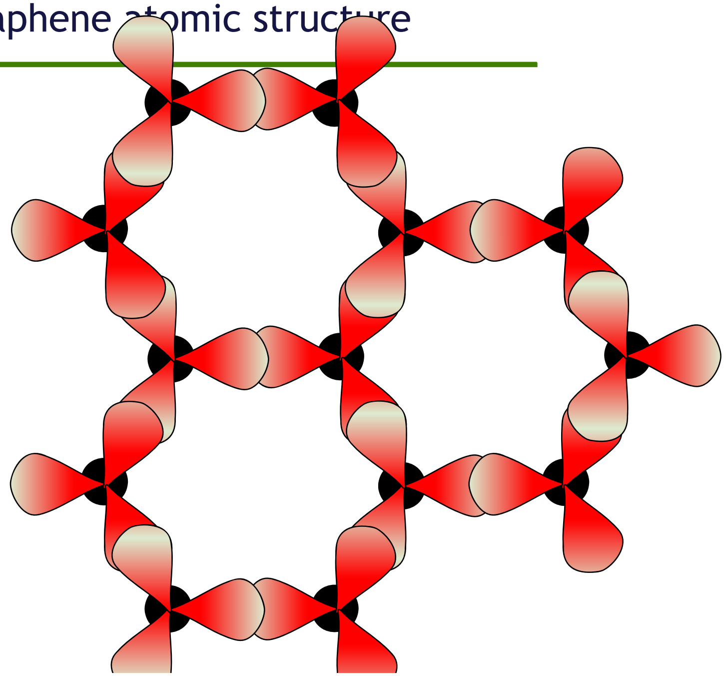


Images by Silvia Milana, Univ. Cambridge (2013)

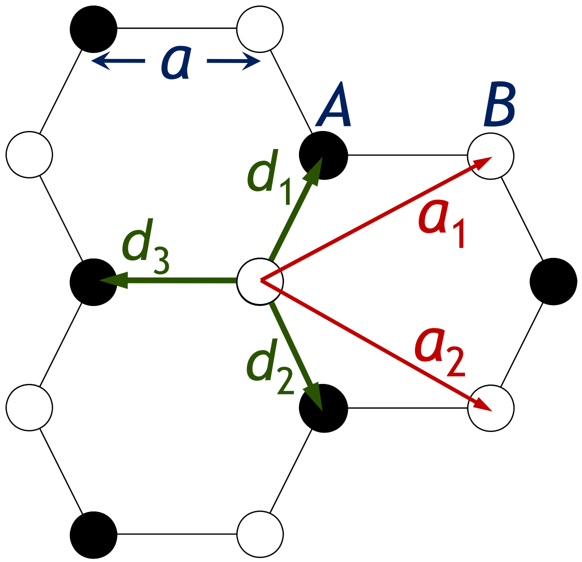
Carbon: $^6C=1s^2 2s^2 2p^2$, group IV



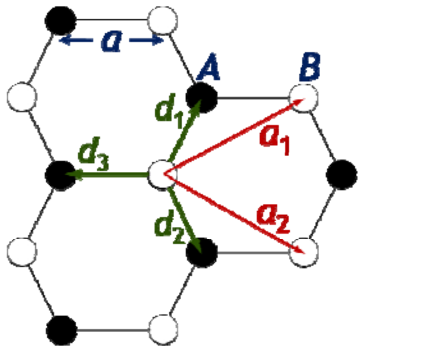
Graphene atomic structure



Graphene electronic structure: p_z orbitals



Graphene: a 2D hexagonal lattice with a basis



nearest neighbor vectors

lattice vectors

$$\mathbf{a}_1 = \frac{a}{2}(3,\sqrt{3})$$

$$\mathbf{a}_1 = \frac{a}{2}(3,-\sqrt{3})$$

$$\mathbf{d}_1 = \frac{a}{2}(1,\sqrt{3})$$

$$\mathbf{d}_2 = \frac{a}{2}(1,-\sqrt{3})$$

$$\mathbf{d}_3 = \frac{a}{2}(-1,0)$$

Schrödinger equation

$$\hat{H}\cdot\Psi(\mathbf{r}) = E\cdot\Psi(\mathbf{r})$$

wavefunction

$$\Psi(\mathbf{r}) = c_A\Phi_A(\mathbf{r}) + c_B\Phi_B(\mathbf{r})$$

$$\Phi_A(\mathbf{r}) = \sum_{\mathbf{r}_A} e^{i\mathbf{k}\cdot\mathbf{r}_A}\phi(\mathbf{r}-\mathbf{r}_A)$$

$$\Phi_B(\mathbf{r}) = \sum_{\mathbf{r}_B} e^{i\mathbf{k}\cdot\mathbf{r}_B}\phi(\mathbf{r}-\mathbf{r}_B)$$

unknowns are the c_A and c_B as function of k

eigenvalue problem

$$\langle\Phi_A|\hat{H}|\Psi\rangle = E\langle\Phi_A|\Psi\rangle$$

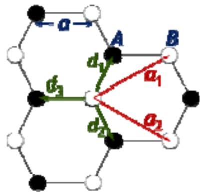
$$\langle\Phi_B|\hat{H}|\Psi\rangle = E\langle\Phi_B|\Psi\rangle$$

Tight-binding approach

- Eigenvalue problem

$$\langle \Phi_B | \hat{H} | \Psi \rangle = E \langle \Phi_B | \Psi \rangle$$
$$\langle \Phi_A | \hat{H} | \Psi \rangle = E \langle \Phi_A | \Psi \rangle$$

$$\Psi(\mathbf{r}) = c_A \Phi_A(\mathbf{r}) + c_B \Phi_B(\mathbf{r})$$
$$\Phi_A(\mathbf{r}) = \sum_{\mathbf{r}_A} e^{i\mathbf{k}\cdot\mathbf{r}_A} \phi(\mathbf{r}-\mathbf{r}_A)$$
$$\Phi_B(\mathbf{r}) = \sum_{\mathbf{r}_B} e^{i\mathbf{k}\cdot\mathbf{r}_B} \phi(\mathbf{r}-\mathbf{r}_B)$$



- Integrals

$$\langle \phi_{\mathbf{r}_A} | \hat{H} | \phi_{\mathbf{r}_B} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_A - \mathbf{r}_B)} = t \cdot e^{i\mathbf{k}\cdot\mathbf{d}}$$ *if $\mathbf{r}_A$ and $\mathbf{r}_B$ are neighbors, otherwise = 0*

$$\langle \phi_{\mathbf{r}_A} | \hat{H} | \phi_{\mathbf{r}_{A'}} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_A - \mathbf{r}_{A'})} = t' \cdot e^{i\mathbf{k}\cdot\mathbf{a}}$$ *if $\mathbf{r}_A$ and $\mathbf{r}_{A'}$ are neighbors, otherwise = 0*

$$\langle \phi_{\mathbf{r}_A} | \phi_{\mathbf{r}_B} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_A - \mathbf{r}_B)} = 0$$

$$\langle \phi_{\mathbf{r}_A} | \phi_{\mathbf{r}_{A'}} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_A - \mathbf{r}_{A'})} = 1$$ *if $\mathbf{r}_A = \mathbf{r}_{A'}$, otherwise = 0*

- We get *t*, *t'* from experiment

$t \approx 3$ eV
$t' \approx 0.1$ eV

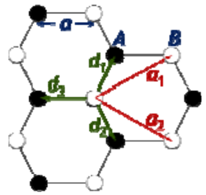
t'<<t, thus we only assume $\mathbf{r}_A \leftrightarrow \mathbf{r}_B$ hopping

Tight-binding approach

- Eigenvalue problem

$$\langle \Phi_A | \hat{H} | \Psi \rangle = E \langle \Phi_A | \Psi \rangle$$
$$\langle \Phi_B | \hat{H} | \Psi \rangle = E \langle \Phi_B | \Psi \rangle$$

$$\Psi(\mathbf{r}) = c_A \Phi_A(\mathbf{r}) + c_B \Phi_B(\mathbf{r})$$
$$\Phi_A(\mathbf{r}) = \sum_{\mathbf{r}_A} e^{i\mathbf{k}\cdot\mathbf{r}_A} \phi(\mathbf{r}-\mathbf{r}_A)$$
$$\Phi_B(\mathbf{r}) = \sum_{\mathbf{r}_B} e^{i\mathbf{k}\cdot\mathbf{r}_B} \phi(\mathbf{r}-\mathbf{r}_B)$$



$$t \cdot \left(e^{i\mathbf{k}\cdot\mathbf{d}_1} + e^{i\mathbf{k}\cdot\mathbf{d}_2} + e^{i\mathbf{k}\cdot\mathbf{d}_3}\right) \cdot c_B = E \cdot c_A$$
$$t \cdot \left(e^{-i\mathbf{k}\cdot\mathbf{d}_1} + e^{-i\mathbf{k}\cdot\mathbf{d}_2} + e^{-i\mathbf{k}\cdot\mathbf{d}_3}\right) \cdot c_A = E \cdot c_B$$

set $f(\mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{d}_1} + e^{i\mathbf{k}\cdot\mathbf{d}_2} + e^{i\mathbf{k}\cdot\mathbf{d}_3}$

$$\begin{pmatrix} 0 & t \cdot f(\mathbf{k}) \\ t \cdot f^*(\mathbf{k}) & 0 \end{pmatrix} \begin{pmatrix} c_A \\ c_B \end{pmatrix} = E \begin{pmatrix} c_A \\ c_B \end{pmatrix}$$

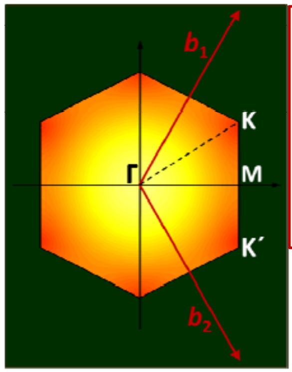
- Solution to the eigenvalue problem

$$\begin{vmatrix} -E & t \cdot f(\mathbf{k}) \\ t \cdot f^*(\mathbf{k}) & -E \end{vmatrix} = 0 \;\Rightarrow\; E = \pm t \cdot |f(\mathbf{k})|$$

$$|f(\mathbf{k})| = \sqrt{3 + 2\cos\left(\sqrt{3}k_y a\right) + 4\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\cos\left(\frac{3}{2}k_x a\right)}$$

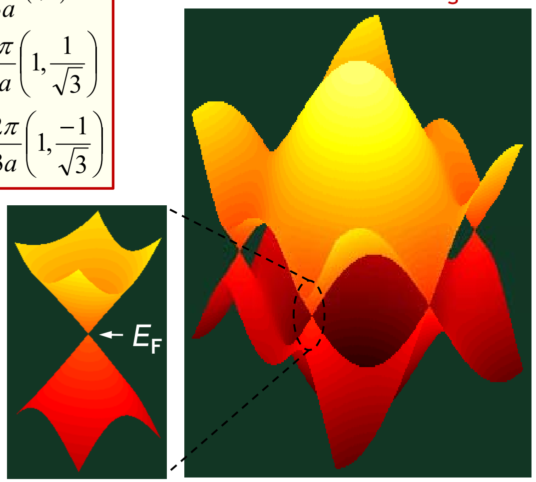
Graphene band structure

- Relation between wavevector **k** (momentum) and energy *E*



$$\mathbf{M} = \frac{2\pi}{3a}(1,0)$$
$$\mathbf{K} = \frac{2\pi}{3a}\left(1, \frac{1}{\sqrt{3}}\right)$$
$$\mathbf{K}' = \frac{2\pi}{3a}\left(1, \frac{-1}{\sqrt{3}}\right)$$

- zero-gap semiconductor
- linear band crossing



Dirac point

- Expand around **K**-point $\quad \mathbf{k} \to \mathbf{K} + \mathbf{q} \quad \mathbf{K} = \frac{2\pi}{3a}\left(1, \frac{1}{\sqrt{3}}\right)$

$$k_x = \frac{2\pi}{3a} + q_x \qquad k_y = \frac{2\pi}{3\sqrt{3}a} + q_y$$

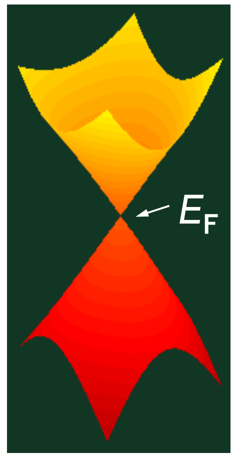
$$|f(\mathbf{k})| \cong \frac{3}{2} a |\mathbf{q}|$$

- Energy around **K**-point

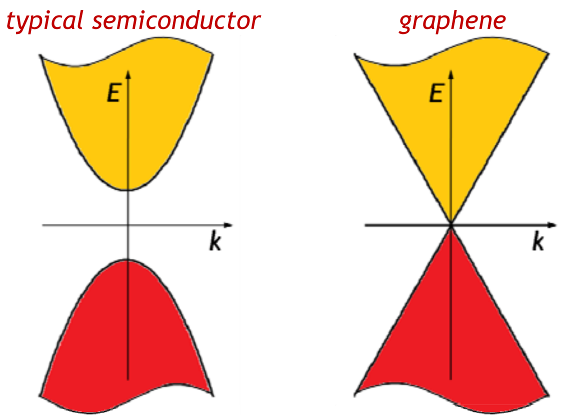
$$E \cong \pm \frac{3}{2} t \cdot a |\mathbf{q}| = \pm \hbar \upsilon_F |\mathbf{q}|$$

$$\upsilon_F = \frac{3}{2}\frac{t \cdot a}{\hbar} \approx 10^6 \text{ m/s}$$

$$E = \sqrt{p^2c^2 + m^2c^4} \qquad \begin{cases} m = 0 \\ c = \upsilon_F \end{cases}$$



Massless Dirac fermions

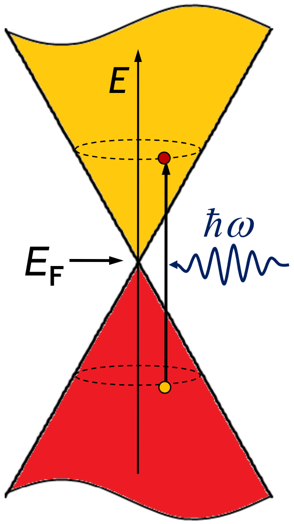


$$f(\mathbf{k}) \cong \frac{3}{2} a(q_x + iq_y) \qquad \hat{H} \cong \begin{pmatrix} 0 & \hbar\upsilon_F(\partial_x + i\partial_y) \\ \hbar\upsilon_F(\partial_x - i\partial_y) & 0 \end{pmatrix} = \hbar\upsilon_F \boldsymbol{\sigma}\cdot\mathbf{p}$$

Optical properties

- Direct optical transitions
  - transition rate (Fermi’s golden rule)

$$\eta = \frac{2\pi}{\hbar} |M|^2 D\left(\frac{\hbar\omega}{2}\right)$$



Dirac density of states

- 2D density of states

$$D(k)dk = 4\frac{2\pi k \cdot dk}{(2\pi)^2}$$

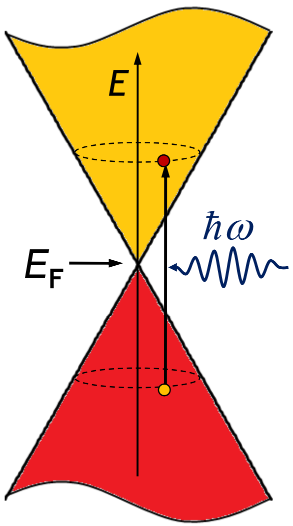
- Change into energy

$$E = \hbar\upsilon_F k$$

$$D(E)dE = \frac{2E \cdot dE}{\pi\hbar^2\upsilon_F^2}$$

- The joint density of states is DOS at $\hbar\omega/2$

$$D\left(\frac{\hbar\omega}{2}\right) = \frac{\hbar\omega}{\pi\hbar^2\upsilon_F^2} = \frac{\omega}{\pi\hbar\upsilon_F^2}$$



Transition matrix element

- Hamiltonian with minimal coupling

$$\hat{H} = \upsilon_F \boldsymbol{\sigma}\cdot\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right) = \hat{H}_0 + \hat{H}'$$

- Vector potential as a function of electric field

$$\mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \Rightarrow \mathbf{A} = i\frac{c\mathbf{E}}{\omega}$$

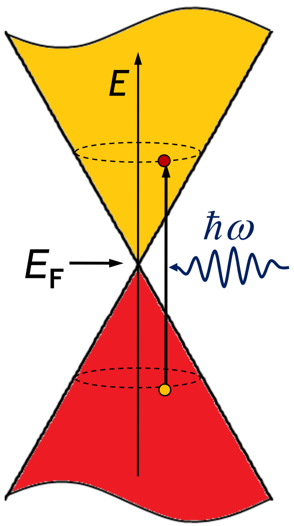
$$\hat{H}' = -i\upsilon_F \frac{e}{\omega} \boldsymbol{\sigma}\cdot\mathbf{E}$$

- Matrix element

$$|M|^2 = |\langle f | \hat{H}' | i \rangle|^2 = \frac{1}{16}\upsilon_F^2 \frac{e^2}{\omega^2} |\mathbf{E}|^2$$

- Electric field as a function of incident power

$$|M|^2 = I\frac{\pi e^2 \upsilon_F^2}{2c\omega^2}$$



Optical properties

- Direct optical transitions
  - transition rate (Fermi's golden rule)

$$\eta = \frac{2\pi}{\hbar} |M|^2 D\left(\frac{\hbar\omega}{2}\right)$$

  - transition matrix element

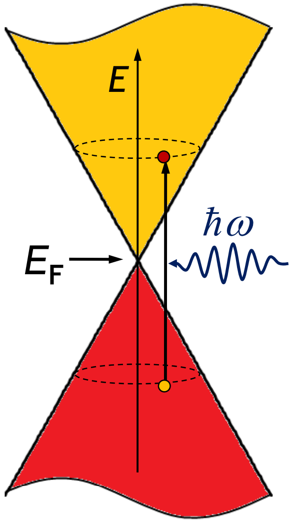
$$|M|^2 = I\frac{\pi e^2 \upsilon_F^2}{2c\omega^2}$$

  - Dirac density of states

$$D\left(\frac{\hbar\omega}{2}\right) = \frac{\omega}{\pi\hbar\upsilon_F^2}$$

  - absorption coefficient

$$A = \frac{\eta\hbar\omega}{I} = \pi\frac{e^2}{\hbar c} = \pi\alpha$$



Universal optical conductance

- Real part of conductivity

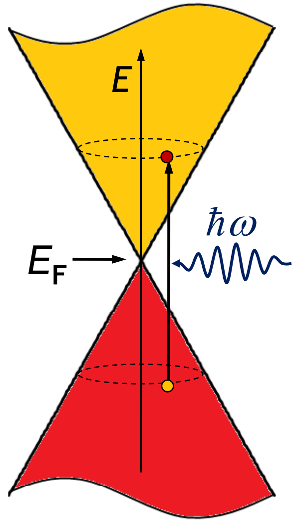
$$\Re\sigma(\omega) = \frac{\pi e^2}{\omega} |\mathbf{v}(\omega)|^2 D(\omega)$$

$$|\mathbf{v}(\omega)|^2 \propto \upsilon_F^2 \qquad D(\omega) \propto \frac{\omega}{\pi\hbar\upsilon_F^2}$$

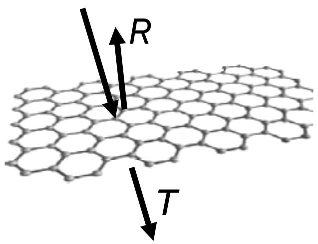
$$\Re\sigma(\omega) = \frac{e^2}{4\hbar}$$

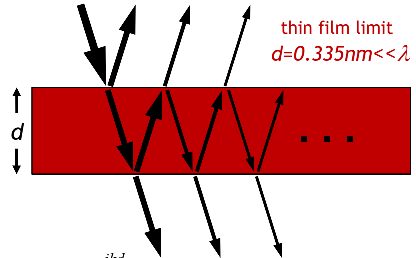
- Dielectric constant

$$\varepsilon(\omega) = 1 + i\frac{4\pi\sigma(\omega)}{\omega d}$$



Graphene optical transmission





thin film limit
d=0.335nm<<λ

$$t = t_{12}e^{ikd}t_{21} + t_{12}e^{ikd}r_{21}e^{ikd}r_{21}e^{ikd}t_{21} + \ldots = \frac{t_{12}t_{21}e^{ikd}}{1 - r_{21}r_{21}e^{i2kd}}$$

Everything is a function of dielectric function ε, and thus of the conductance σ

Light transmission in the thin film limit

$$T = |t|^2 \approx 1 - \frac{4\pi}{c}\Re\sigma = 1 - \pi\frac{e^2}{\hbar c} = 1 - \pi\alpha$$

$$\alpha \equiv \frac{e^2}{\hbar c}$$ fine structure constant

Light reflection in the thin film limit

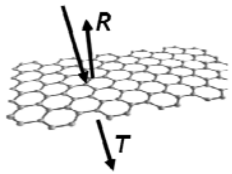
$$R \propto \omega^2 d^2 \approx 0$$

Light absorption = 1-R-T = $\pi\alpha$

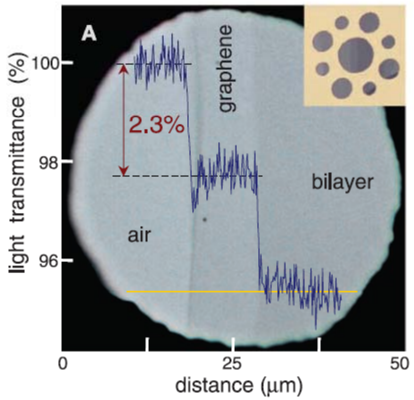
- universal absorption $\pi\alpha$ per graphene layer

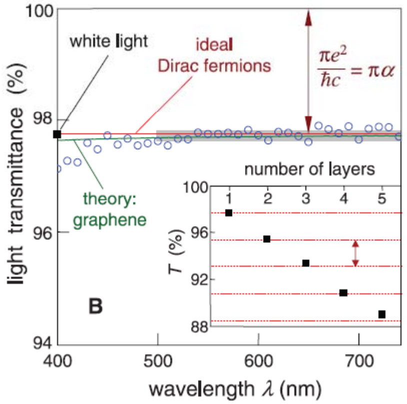
$$\pi\alpha = \frac{3.14159\ldots}{137.035\ldots} \approx 0.023 = 2.3\%$$

Graphene optical transmission



- Experimental verification of $\pi\alpha$ absorption per layer
  - experiments on graphene suspended in air

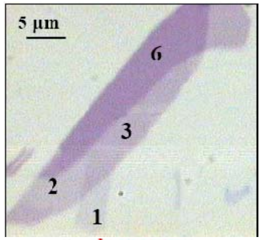


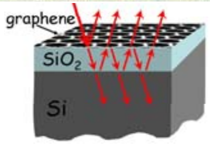


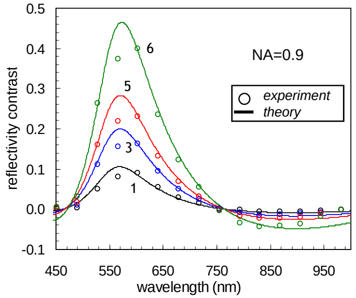
Nair et al., Science 320, 1308 (2008)

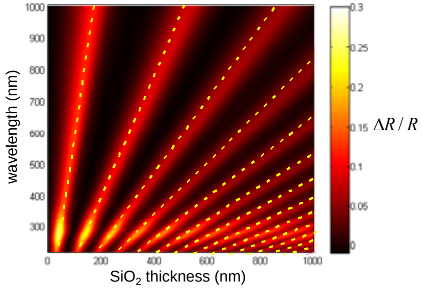
Graphene visibility

- If graphene does not reflect, how come it is visible?
  - only in specific substrates due to interference
  - choose a substrate with small initial reflection
  - graphene's effect is then maximal
- Condition for small reflection: antireflection
  - spacer thickness = $\lambda/4$
  - spacer index = $\sqrt{\text{substrate index}}$
- Experimental verification





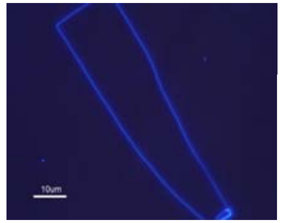


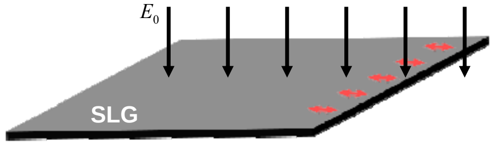


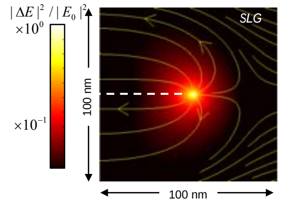
Casiraghi et al., Nano Letters 7, 2711 (2007)

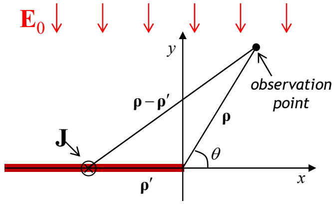
Dark field visibility

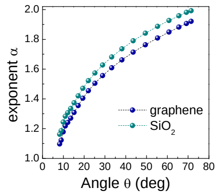
- In dark field mode, only the edges of graphene "light up"
  - Numerical simulations and theory predict this behavior

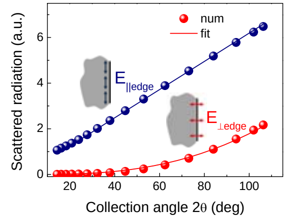












Rayleigh scattering in the "flatland"

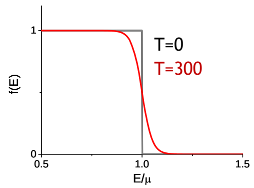
$$I(\theta)\propto k^{\alpha}\rho^{\alpha-2}t^{2}\,|\varepsilon_r-1|^{2}\sin^{2}\theta$$

Conductance at finite temperature and doping

- Fermi-Dirac distribution function

$$f(E)=\frac{1}{e^{(E-\mu)/k_BT}+1}$$

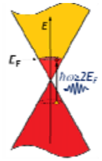
μ: chemical potential (due to doping)
T: temperature (K)
k_B: Boltzmann constant



- Interband transitions in the Dirac cone

$$\Re\sigma_{inter}(\omega)=\frac{\pi e^2}{\omega}|\mathbf{v}(\omega)|^2\,D(\omega)\left[f\left(-\frac{\hbar\omega}{2}\right)-f\left(\frac{\hbar\omega}{2}\right)\right]$$

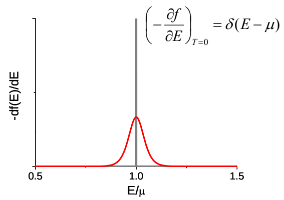
$$\approx\frac{e^2}{8\hbar}\left[\tanh\left(\frac{\hbar\omega+2\mu}{4k_BT}\right)+\tanh\left(\frac{\hbar\omega-2\mu}{4k_BT}\right)\right]$$



- Intraband transitions of free carriers

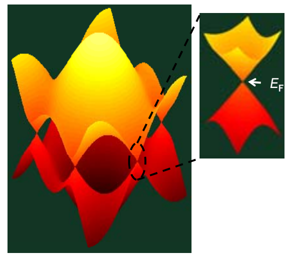
$$\sigma_{\text{intra}}(\omega)=e^2\int 4\frac{d\mathbf{k}}{4\pi^2}\frac{\mathbf{v}(\mathbf{k})\cdot\mathbf{v}(\mathbf{k})}{1/\tau-i\omega}\left(-\frac{\partial f}{\partial E}\right)_{E=E(\mathbf{k})}$$

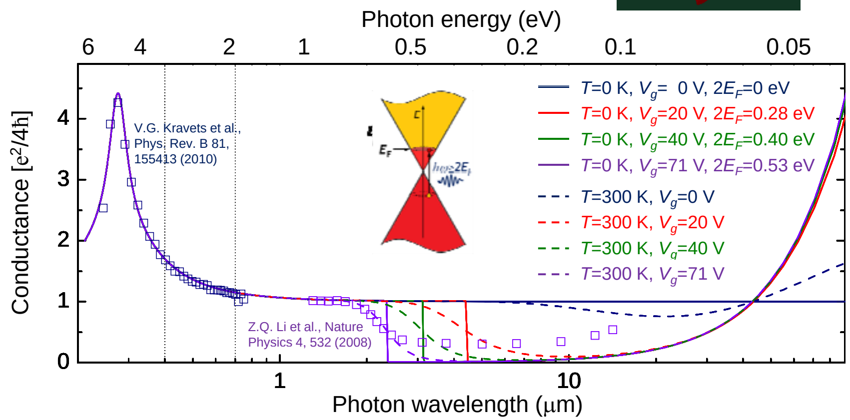
$$=e^2\frac{2k_BT}{\pi\hbar^2}\ln\left(2\cosh\frac{\mu}{2k_BT}\right)\frac{1}{1/\tau-i\omega}$$



Graphene conductance

- The full spectrum of graphene
  - Van Hove peak in UV
  - Universal conductance in VIS
  - Pauli blocking due to doping in NIR
  - Drude free electron conductance in IR





Electric field effect: doping by gating

- Graphene/oxide/semiconductor system
  - Conductor/dielectric/conductor ⇒ capacitor
  - Field applied by bottom gating with voltage V_g

$$Q = C_g \cdot V_g \Rightarrow n = C_g \cdot V_g / e$$

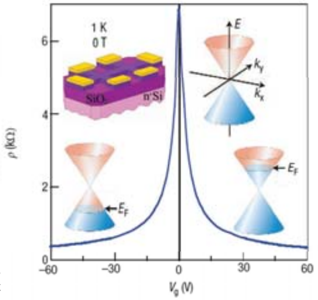
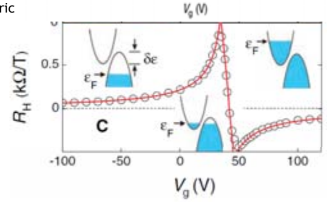
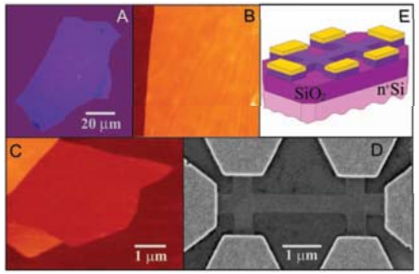
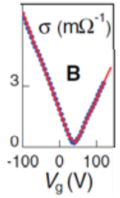
  - The scaled capacitance C_g/e was measured for graphene

$$C_g / e_{(300\text{nmSiO2/Si})} = 7.2\times10^{10}\ \text{cm}^{-1}\text{V}^{-1}$$

  - So we can use

$$n\left[10^{10}\ \text{cm}^{-2}\right] = 7.2\times\frac{300}{t[\text{nm}]}\frac{\varepsilon_r}{3.9}V_g[\text{V}]$$

t: spacer thickness
ε_r: spacer dielecric constant

K.S. Novoselov et al., Science 306, 666 (2004)
A.K. Geim and K.S. Novoselov, Nat. Mater. 6 (2007)

Electric field effect: Fermi level shift

- Number of charges in Fermi level (at T=0)

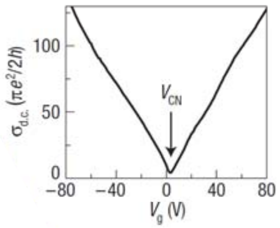
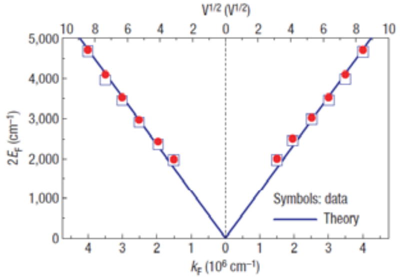
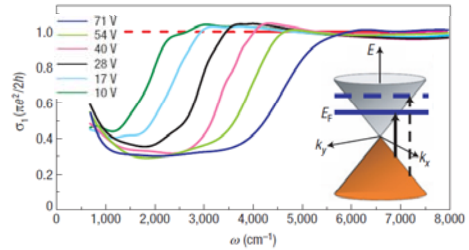
$$n = g_s g_v \int_0^{k_F} \frac{d\mathbf{k}}{4\pi^2} = \frac{4}{4\pi^2}\int_0^{k_F} 2\pi k dk = \frac{k_F^2}{\pi}$$

  - So we can relate the Fermi energy to doping (at T=0)

$$n = k_F^2/\pi = C_g \cdot V_g / e \Rightarrow n \propto V_g$$

$$k_F = \sqrt{\pi n} = \sqrt{\pi C_g V_g / e} \Rightarrow k_F \propto \sqrt{V_g}$$

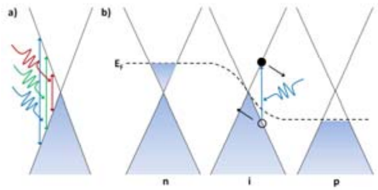
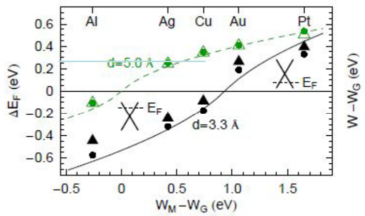
$$E_F = \hbar\upsilon_F\sqrt{\pi n} = \hbar\upsilon_F\sqrt{\pi C_g V_g / e} \Rightarrow E_F \propto \sqrt{V_g}$$

Z.Q. Li et al., Nature Physics 4, 532 (2008)

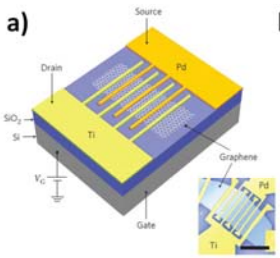
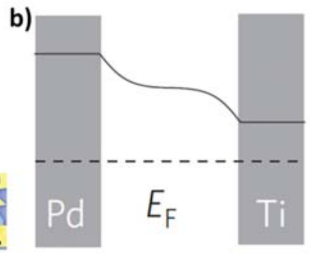
Doping by direct contact

- Graphene contacted by a metal with different Fermi level
  - Local shift of graphene's Fermi level
  - Creation of a lateral "Schottky" junction

G . Giovannetti, et al., Phys. Rev. Lett. 101, 026803 (2008)

- Graphene photodetectors

T . Mueller et al., Nat. Photonics 4, 297 (2010)

Graphene's free charges

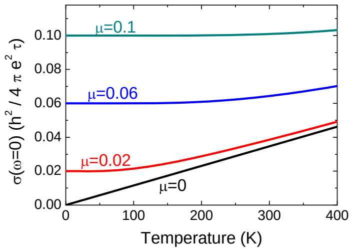
- The free carrier conductance

$$\sigma_{intra} = e^2\frac{2k_BT}{\pi\hbar^2}\ln\left(2\cosh\frac{\mu}{2k_BT}\right)\frac{1}{1/\tau - i\omega}$$

  - At high doping (large Fermi energy E_F or chemical potential μ), the free carrier conductance becomes temperature independent. So we use the T=0 approximation:

$$\sigma_{\text{intra}}(\omega) \approx \frac{e^2 E_F}{\pi\hbar^2}\frac{1}{1/\tau - i\omega}$$

t: free carrier relaxation time

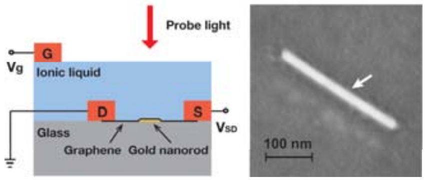


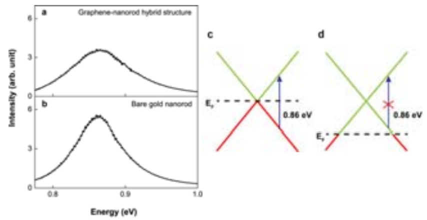
- These carriers have an effective mass $E_F = \upsilon_F p_F = m^*\upsilon_F^2 \Rightarrow m^* = E_F/\upsilon_F^2$
  - The mobility μ^* is related to the relaxation time τ from the Drude formula: $\mu^* = \frac{e\tau}{m^*} \Rightarrow \tau = \frac{\mu^* E_F}{e\upsilon_F^2}$
  - Typical mobility values $\mu^* > 10000\ \text{cm}^2/\text{V}\cdot\text{s}$ or higher. For E_F=0.1 eV we get $\tau > 10^{-13}$ s

    typical values for noble metals are $\tau \approx 10^{-14}$ s
  - *Graphene very promising for IR plasmonics!!*

Graphene plasmonic applications

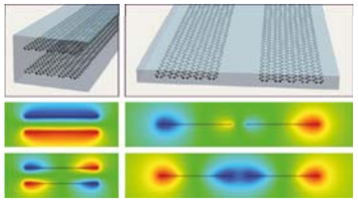
Active plasmonics:
dynamically modulated metal plasmonic





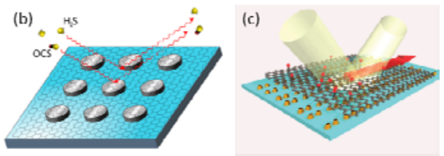
J. Kim et al., *Nano Lett.* 12, 5598 (2012)

Graphene plasmonics:
Waveguides and circuits



J. Christensen et al., ACS Nano 6, 431 (2012)

In plasmonic biosensing:
bio-functionalization and dynamic modulation

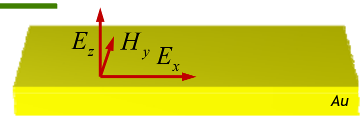


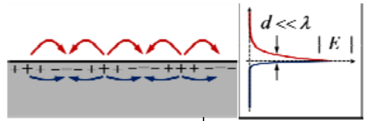
J.C. Reed et al., *Nano Lett.*, 2012, 12, 4090-4094.
V.G. Kravetset al., *Nature Mater.*, 2013, DOI:10.1038/NMAT3537.

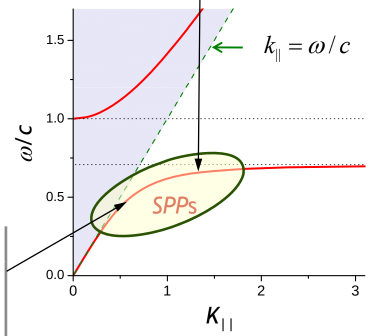
Surface Plasmon Polaritons

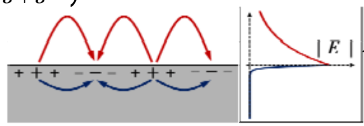
- Charge density oscillations coupled with light
  - Bound to propagate along the metal surface
- General form of all fields
  - above the interface (insulator with dielectric function ε)
    $E_z, E_x, H_y \propto e^{i(k_\parallel x-\omega t)} e^{-\kappa z}$ $\kappa = \sqrt{k_\parallel^2 - \varepsilon\omega^2/c^2}$
  - below the interface (metal with dielectric function ε')
    $E'_z, E'_x, H'_y \propto e^{i(k_\parallel x-\omega t)} e^{+\kappa' z}$ $\kappa' = \sqrt{k_\parallel^2 - \varepsilon'\omega^2/c^2}$
- Boundary conditions
  $E'_x = E_x \qquad H'_x = H_x$
- Surface Plasmon Polariton (SPP)

$$\left.\begin{array}{l}\dfrac{\kappa}{\varepsilon} + \dfrac{\kappa'}{\varepsilon'} = 0 \\ k_\parallel = \dfrac{\omega}{c}\sqrt{\dfrac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}}\end{array}\right\} \quad \varepsilon' < -\varepsilon$$



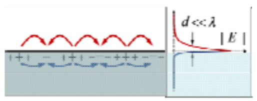


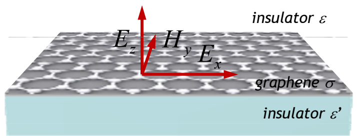




Plasmons on graphene

- Light coupled with Dirac charge density oscillations
  - Bound to propagate along the graphene surface
- General form of all fields
  - above graphene (insulator with dielectric function ε)
    $E_z, E_x, H_y \propto e^{i(k_\parallel x-\omega t)} e^{-\kappa z}$ $\kappa = \sqrt{k_\parallel^2 - \varepsilon\omega^2/c^2}$
  - below graphene (insulator with dielectric function ε')
    $E'_z, E'_x, H'_y \propto e^{i(k_\parallel x-\omega t)} e^{+\kappa' z}$ $\kappa' = \sqrt{k_\parallel^2 - \varepsilon'\omega^2/c^2}$
  - inside graphene (which has conductance σ) we have a current
    $I_x = \sigma E_x$
- Boundary conditions
  $E'_x = E_x \qquad H'_x = H_x + I_x$
- General SPP condition $\dfrac{\kappa}{\varepsilon} + \dfrac{\kappa'}{\varepsilon'} = \dfrac{-i\sigma}{\varepsilon_0\omega}$
  - If $k_\parallel >> \varepsilon\omega^2/c^2 \Rightarrow \kappa = \kappa' \approx k_\parallel$ then $k_\parallel \approx \varepsilon_0 \dfrac{\varepsilon+\varepsilon'}{2}\dfrac{2i\omega}{\sigma}$





using $\varepsilon_g \approx 1 + \dfrac{i\sigma}{d\varepsilon_0\omega}$

we also get

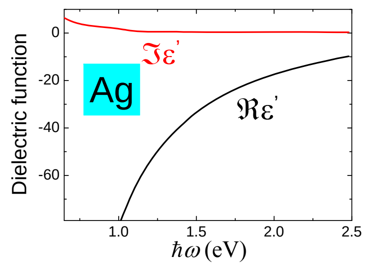
$$\frac{\kappa}{\varepsilon} + \frac{\kappa'}{\varepsilon'} = \frac{1-\varepsilon_g}{d}$$
$$k_\parallel \approx \frac{\varepsilon+\varepsilon'}{d(1-\varepsilon_g)}$$

SPP conditions

- We get two different SPP relations

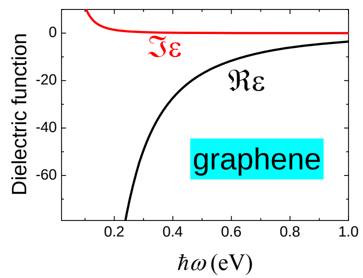
typical metal/dielectric SPP

$$\frac{\kappa}{\varepsilon} + \frac{\kappa'}{\varepsilon'} = 0$$
$$k_\parallel = \frac{\omega}{c}\sqrt{\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}}$$

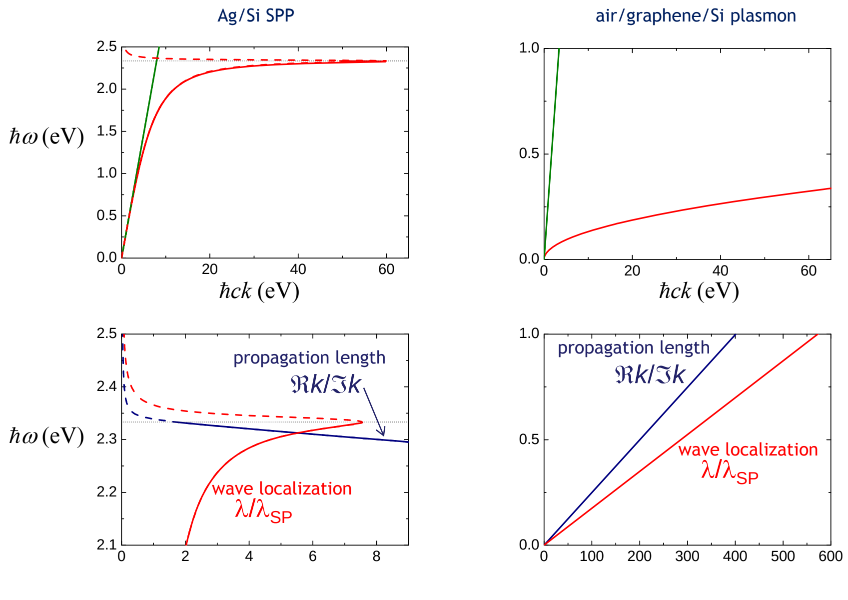


typical dielectric/graphene/dielectric plasmon

$$\frac{\kappa}{\varepsilon} + \frac{\kappa'}{\varepsilon'} = \frac{1-\varepsilon_g}{d}$$
$$k_\parallel \approx \frac{\varepsilon+\varepsilon'}{d(1-\varepsilon_g)}$$

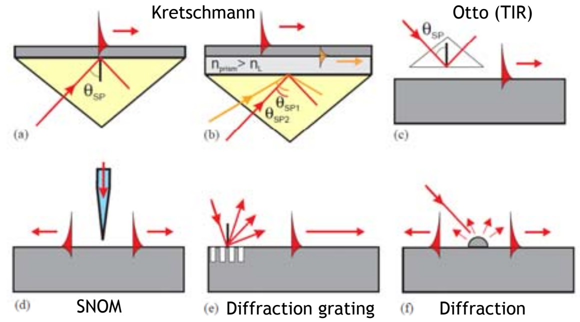


Camparison between SPPs



Excitation of SPPs

- Surface plasmon polaritons exist below the light-cone
  - momentum conservation does not allow direct excitation
  - special care is needed to excite them

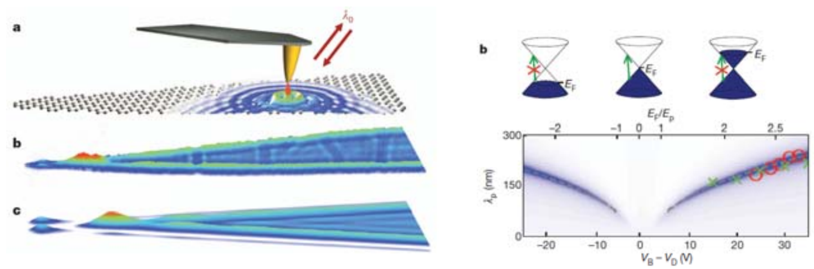


Zayats, Smolyaninov and Maradudin, Phys. Rep. 408, 131 (2005)

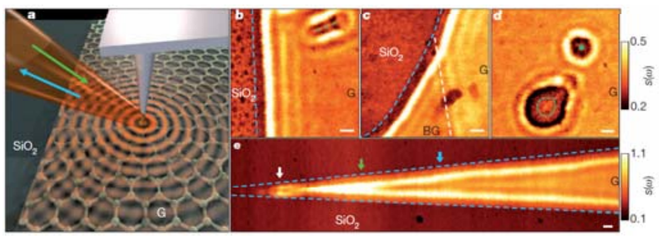
- In grating excitation:

$$k_{\parallel} \to k_{\parallel} + m\frac{2\pi}{a}, \quad m = 0, \pm 1, \pm 2, \ldots$$

Measuring graphene plasmons



J. Chen et al., *Nature* 487, 77 (2012)



Z. Fei et al., *Nature* 487, 82 (2012)